Using Spherical Centroidal Voronoi Tessellations in Climate System Modeling

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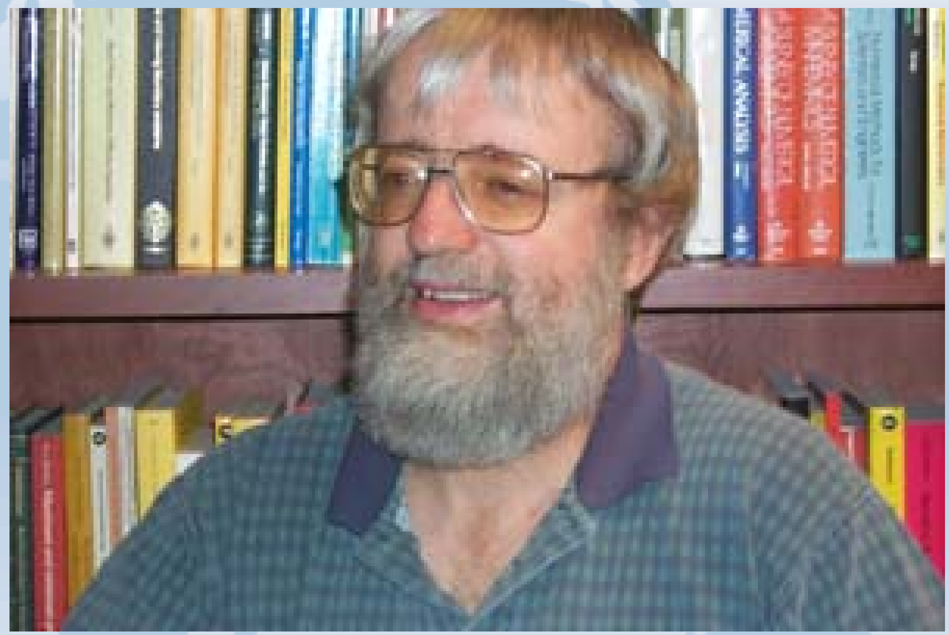
A special thanks ....

Lili Ju



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Outline

The present state: IPCC-class ocean models are built on, at best, quasi-uniform grids.

The problem: Quasi-uniform grids make it difficult to using eddy-permitting resolutions in simulations of order of ~100 years.

A possible answer: SCVT offer a low-risk solution to this problem.

An aside ....

Most of the discussion here will focus on ocean modeling, but we are applying these same techniques to ice sheet modeling (and other climate system components).

I will show an SCVT grid for Greenland at the end.

What does a typical IPCC-class ocean models look like?

The short answer is vintage 1970s.

The models are ....
- built on a quasi-uniform grid.
- built on a structured grid.
- built with finite-difference / finite-volume algorithms.
- built with time splitting of barotropic / baroclinic mode.

The goal is ....
- to conserve some "mass-like" quantity.
- to have an associated tracer equation that is compatible.
- to strive for energy and/or enstrophy consistency.
- to be computationally efficient.

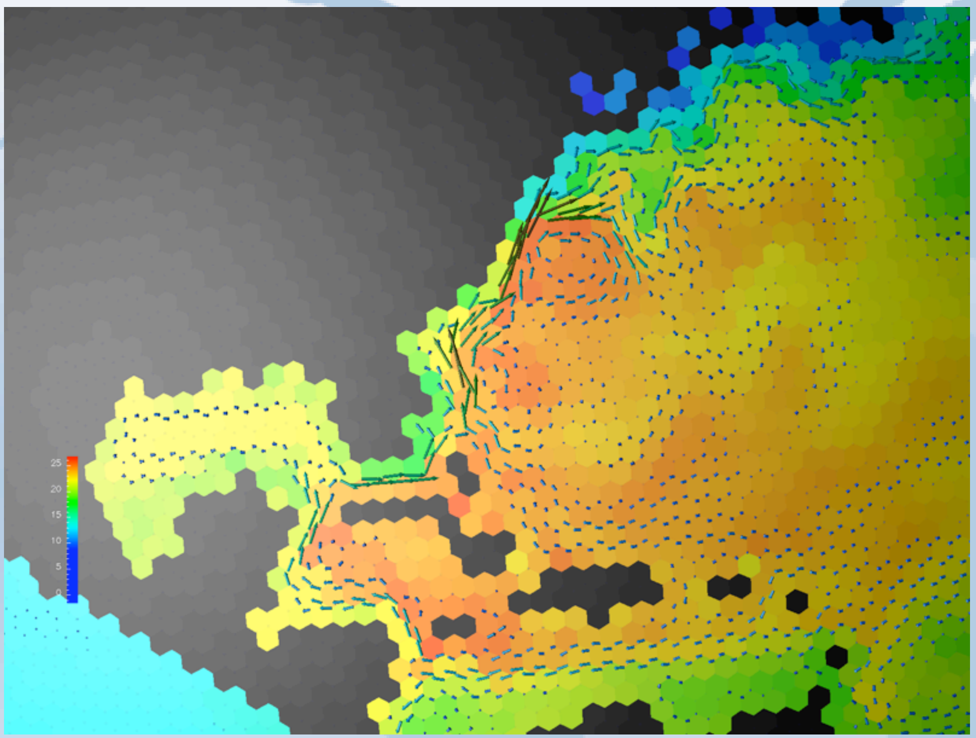
IPCC 4AR Models

BCCR, Norway
CCCma, Canada
CCSR/NIES/FRCGC (hi-res), Japan
CCSR/NIES/FRCGC (med-res), Japan
CNRM, France
CSIRO, Australia
GFDL (CM2.0), USA
GFDL (CM2.1), USA
GISS (C4x3), USA
GISS (Model E-H), USA
GISS (Model E-R), USA
IAP, China
INM, Russia
IPSL, France
MPI, Germany
MRI, Japan
NCAR (CCSM3), USA
NCAR (PCM1), USA
NCC, China
UKMO (HadCM3), UK
UKMO (HadGEM1), UK

A quasi-uniform, IPCC-class, ocean grid.

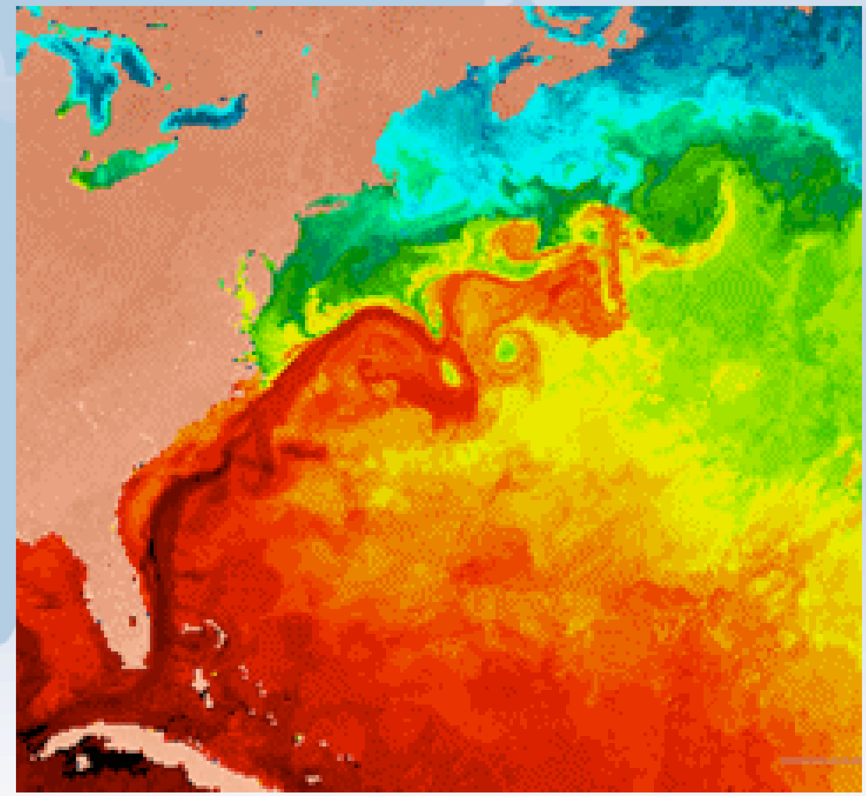
LANL Parallel Ocean Program (POP)

Typical IPCC-class resolution is ~100 km



This is typical of what we get ....

... and this is what we want.



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So what is the issue here?

We are becoming increasingly interested in the understanding abrupt climate change, i.e. those event that are associated with high degrees of nonlinearity and/or critical thresholds.

If our ocean climate simulations lack the nonlinear activity inherent in the real ocean, then there is room for doubt in regards to their utility in understanding abrupt events.

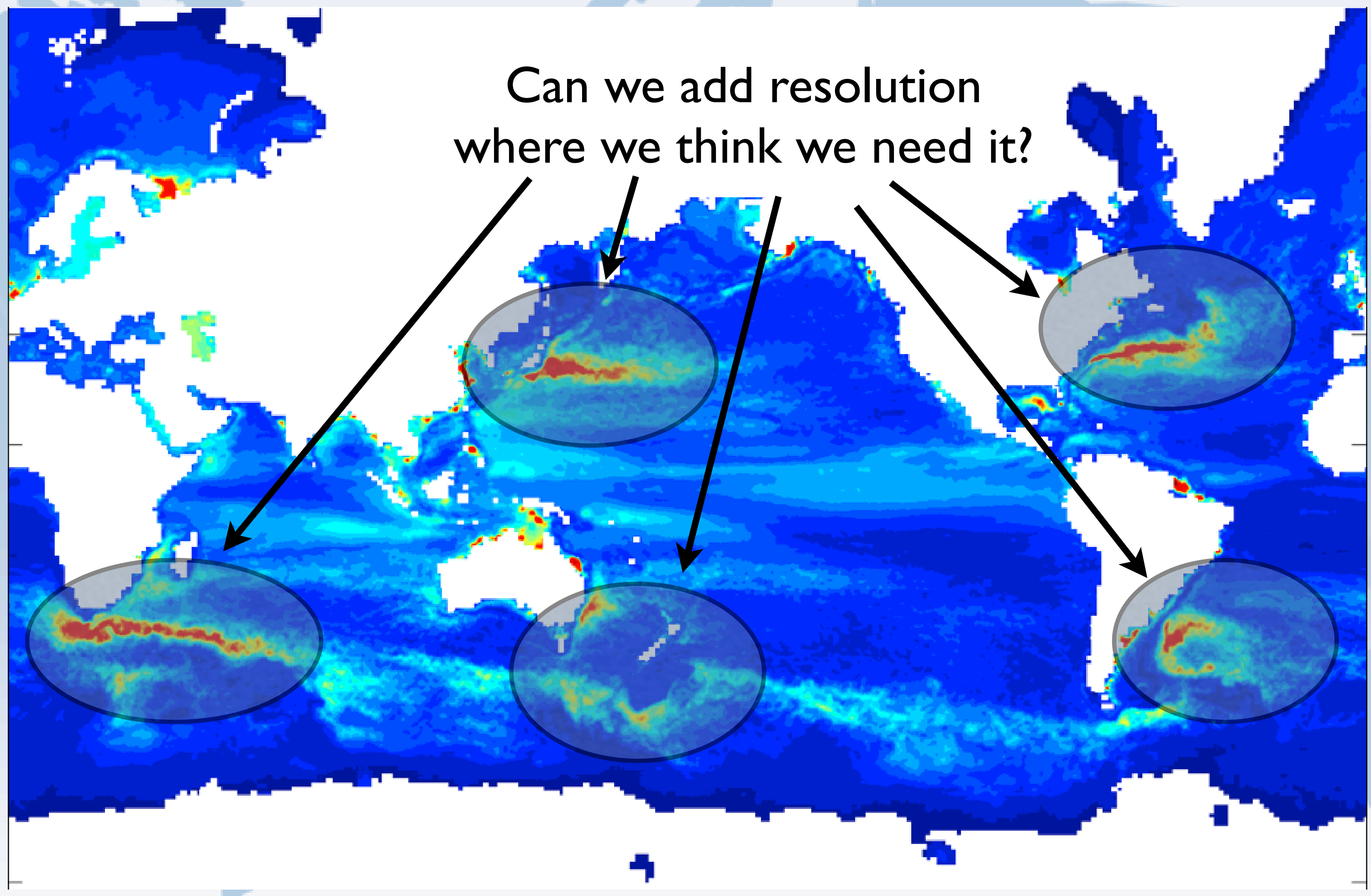
So the goal is to capture this nonlinear (eddy) activity in our IPCC-class simulations.

If we are trying to get from here (coarse-grain simulations) to there (eddy-resolving simulations) with quasi-uniform grids, the numbers don't look good.

A typical IPCC runs: 380x320 w/ 60 minute dt
Eddy resolving simulations: 3600x2400 w/ 6 minute dt

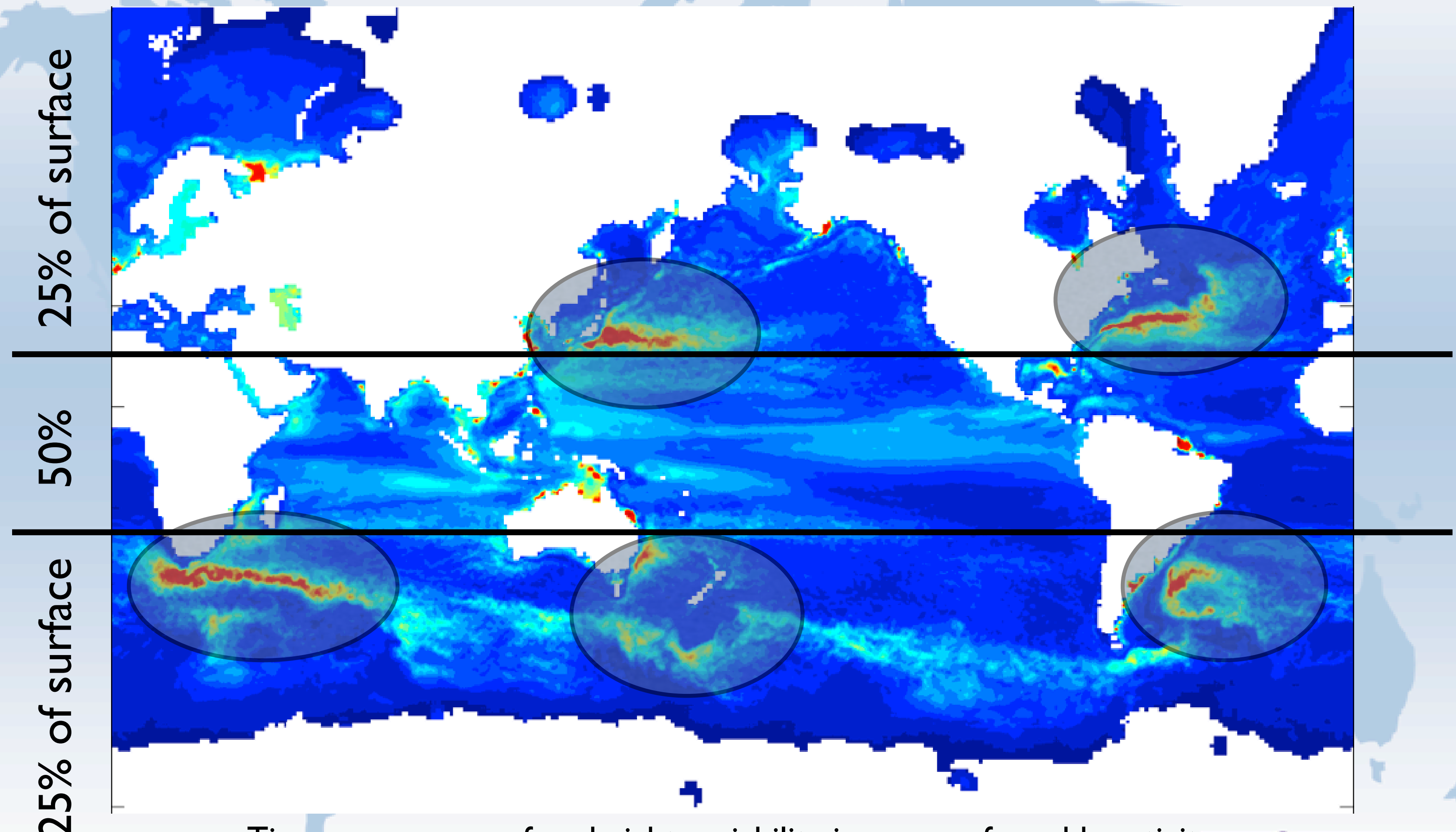
Getting from here to there implies a factor of 1000 in computing burden, or about 10 doubling, or about 15 to 20 years under computing BAU scenarios.

For ocean modeling, there might we another path to eddy-resolving simulations.



Time-mean sea-surface height variability is a proxy for eddy activity.

Are really going to save anything by placing resolution in all of these (and other) places?



Time-mean sea-surface height variability is a proxy for eddy activity.

Our goal here is really quite modest ...

Recall the description of our vintage 1970s models.

The models are ....
- built on a quasi-uniform grid. →
- built on a structured grid. →
- built with finite-volume algorithms.
- built with time splitting of mode.

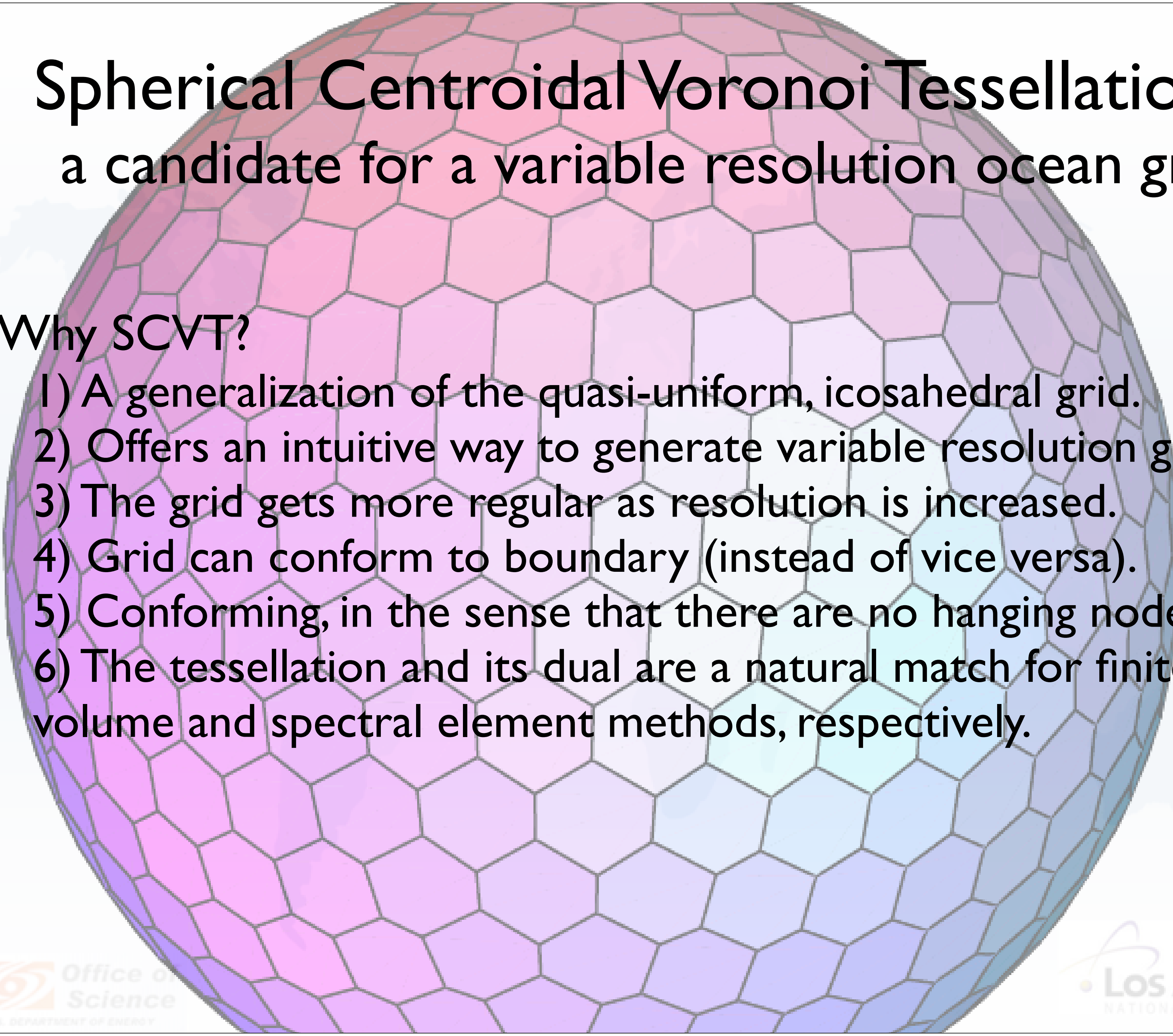
The goal is ....
- conserve mass.
- compatible tracer equation.
- energy and enstrophy consistency.
- to be computationally efficient.

Now updated to 2010 ....

The models are ....
- built w/ user-defined proxy for resolution.
- built on an UNstructured grid.
- built with finite-volume algorithms.
- built with time splitting of mode.

The goal is ....
- conserve mass.
- compatible tracer equation.
- energy and enstrophy consistency.
- to be computationally efficient.

Note that the former is a subset of the latter.



Spherical Centroidal Voronoi Tessellations:
a candidate for a variable resolution ocean grid

Why SCVT?

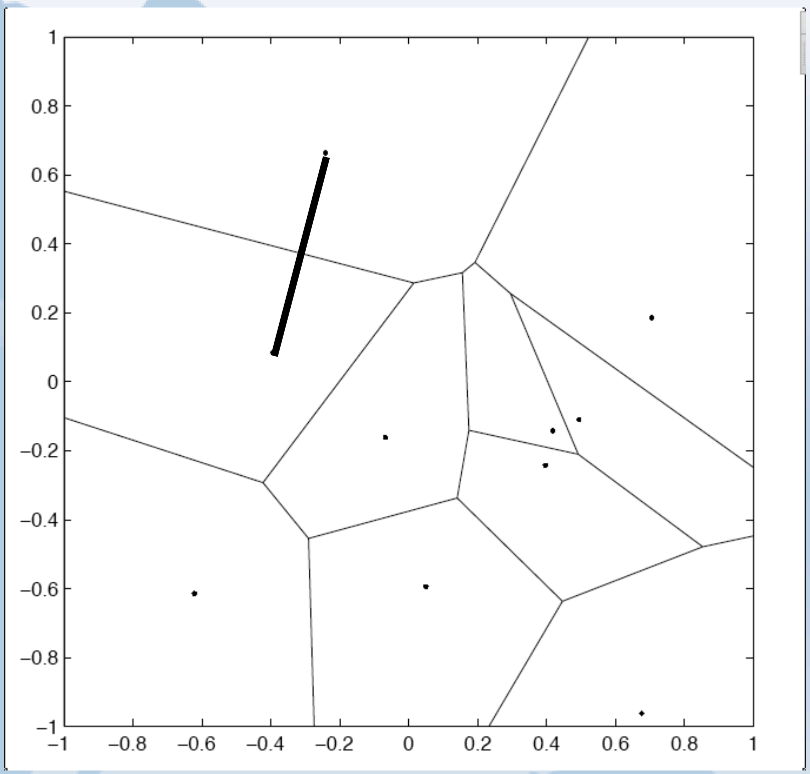
1) A generalization of the quasi-uniform, icosahedral grid.
2) Offers an intuitive way to generate variable resolution grids.
3) The grid gets more regular as resolution is increased.
4) Grid can conform to boundary (instead of vice versa).
5) Conforming, in the sense that there are no hanging nodes.
6) The tessellation and its dual are a natural match for finite-volume and spectral element methods, respectively.

Definition of a Voronoi Tessellations

Given a region, S
And a set of generators, z_i ...

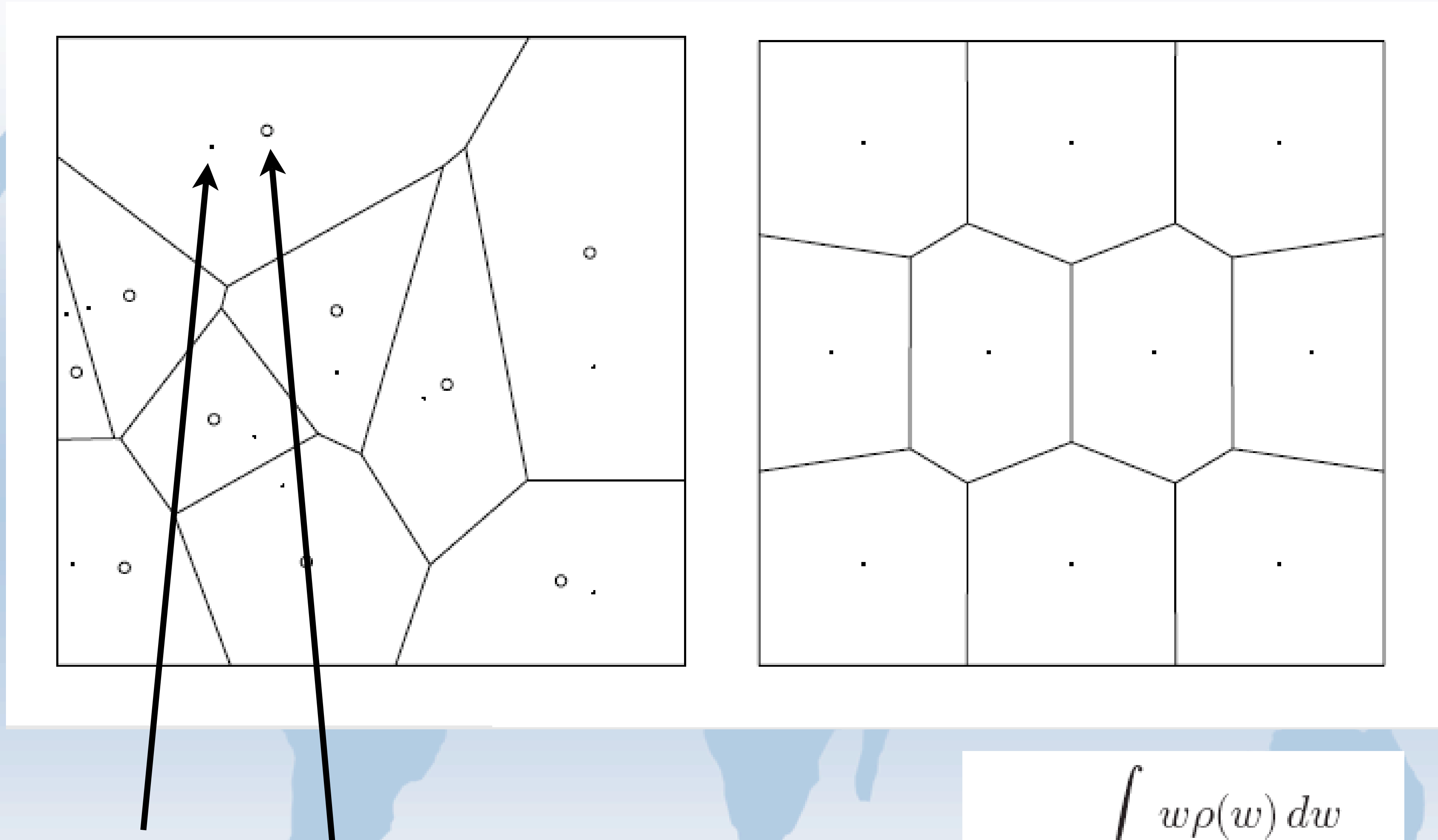
The Voronoi region, V_i, for each z_i is the set of all points closer to z_i than z_j for j not equal to i.

We are guaranteed that the line connecting generators is orthogonal to the shared edge and is bisected by that edge.



But this does not mean that the grid is nice ....

Definition of a Centroidal Voronoi Tessellations



z_i

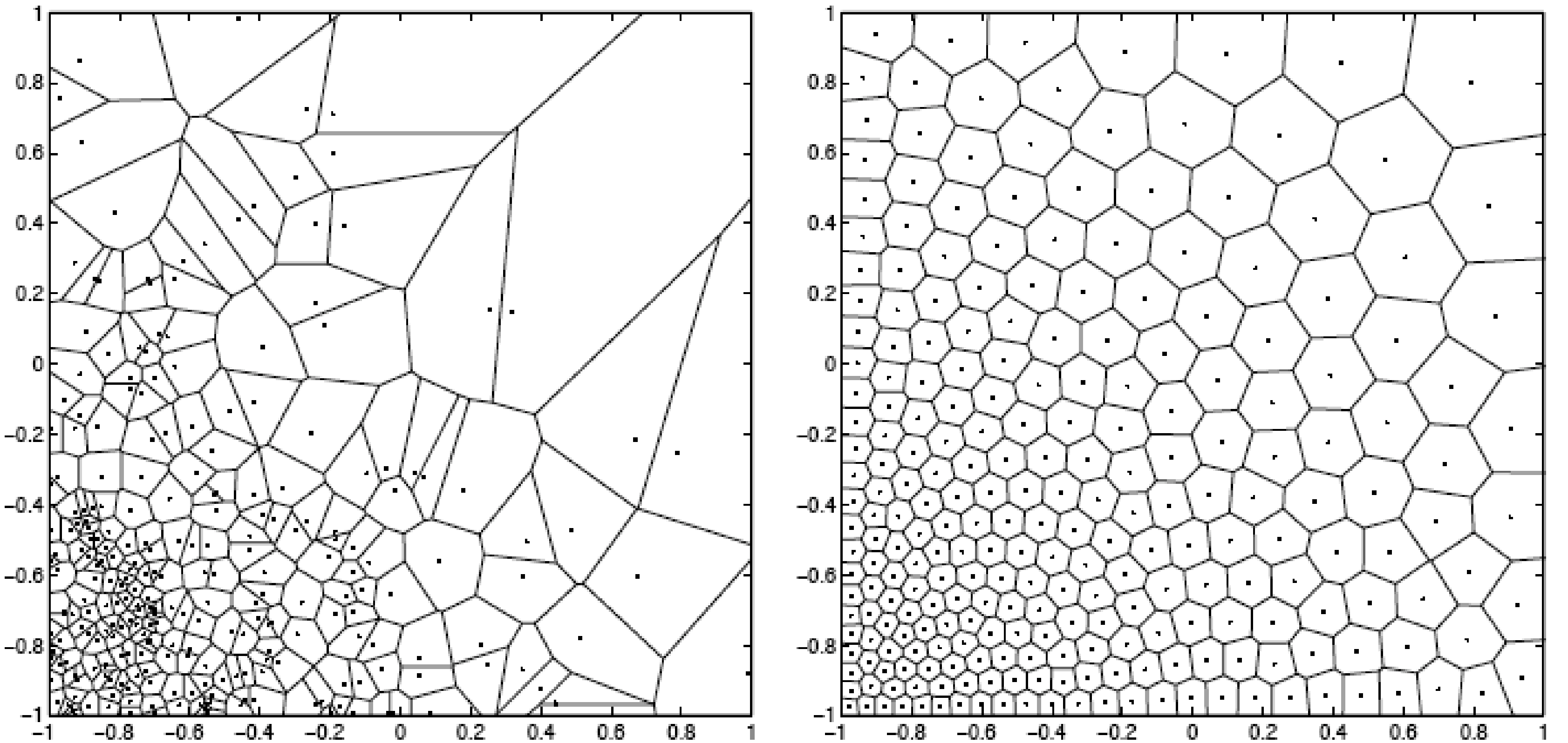
z_i^* = center of mass wrt a user-defined density function

$$z^* = \frac{\int_V w\rho(w)\,dw}{\int_V \rho(w)\,dw}$$

Non-uniform Centroidal Voronoi Tessellations

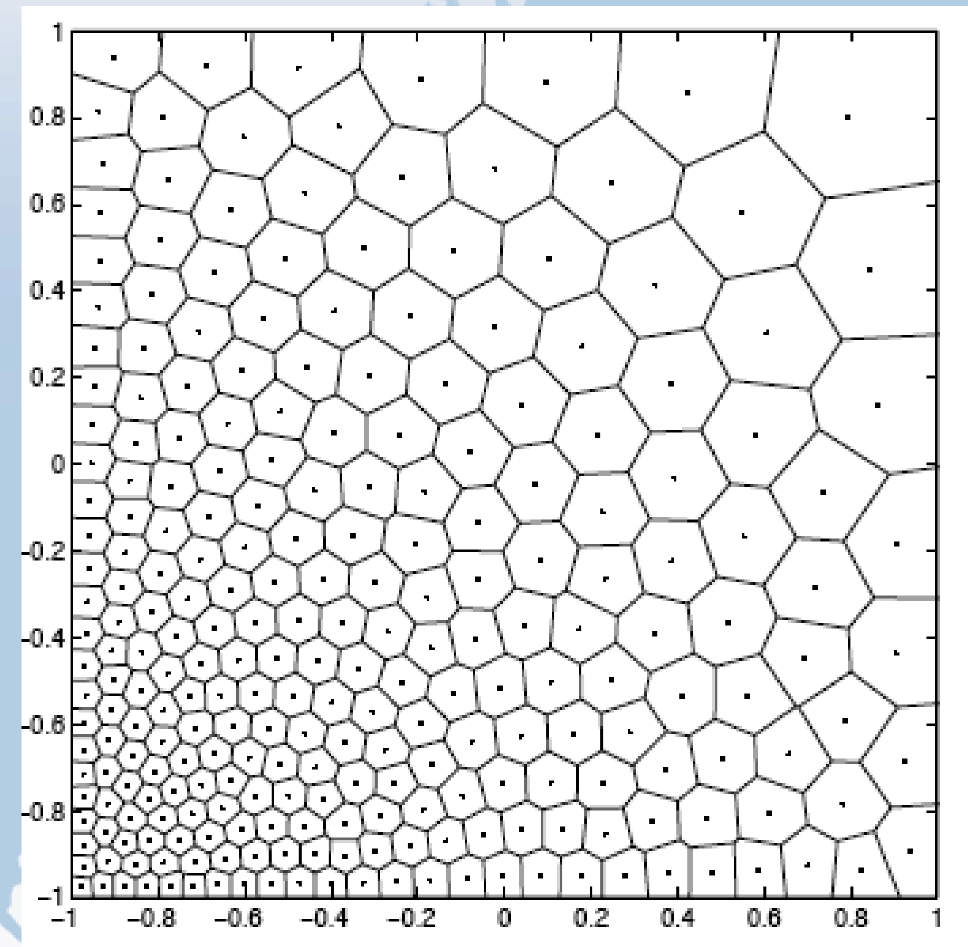
Distribute generators in such a way as to make the grid regular.

Also biases the location of those generators to regions of high density.



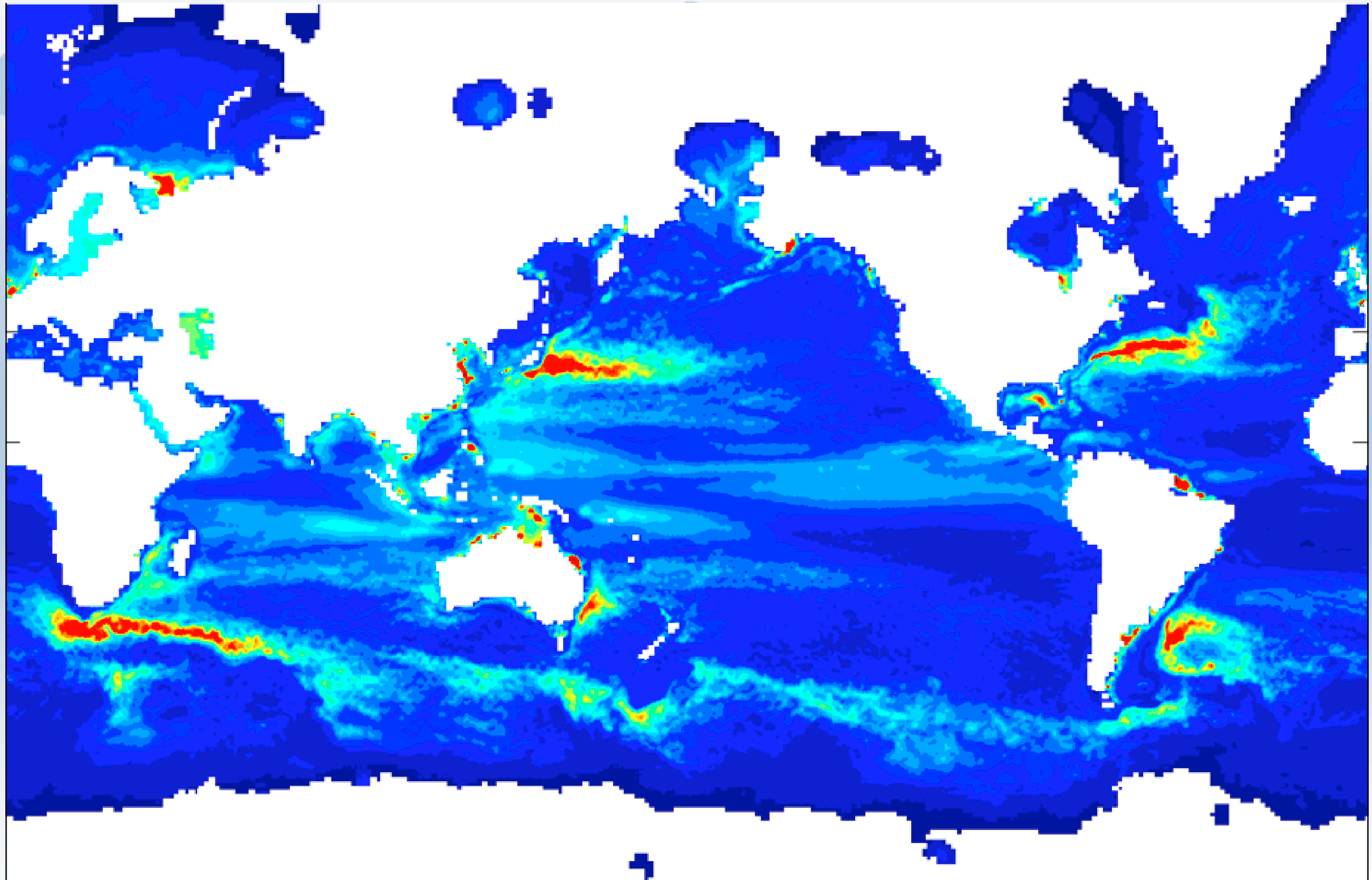
Random sampling | Centroidal Voronoi

Gersho conjecture (now proven in 2D) tells us that as we added generators, all cells evolve toward perfect hexagons. Meaning that the grid just keeps getting more regular as we add resolution.



Recall our proxy for eddy activity ....

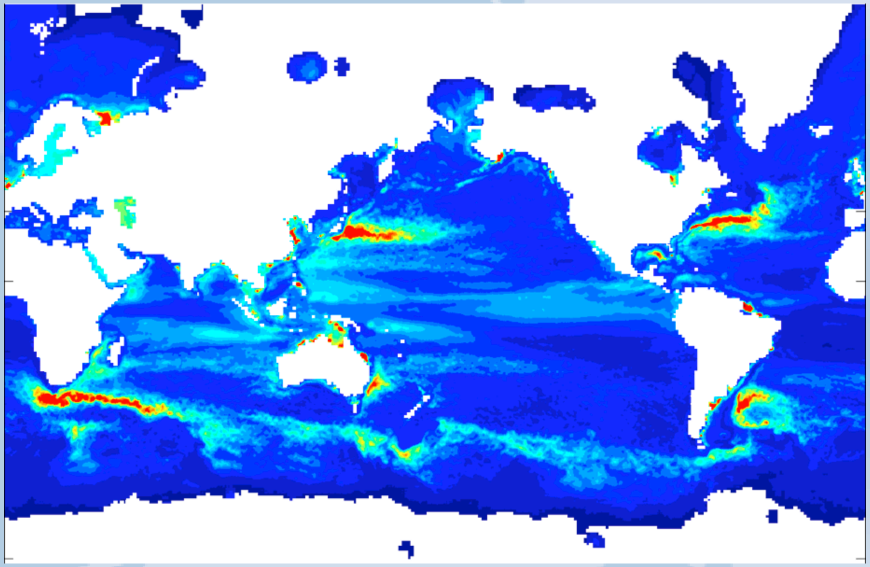
this can also be a proxy for where we want additional resolution



So how do we generate these grids?

1) seed the domain with 163842 generators.

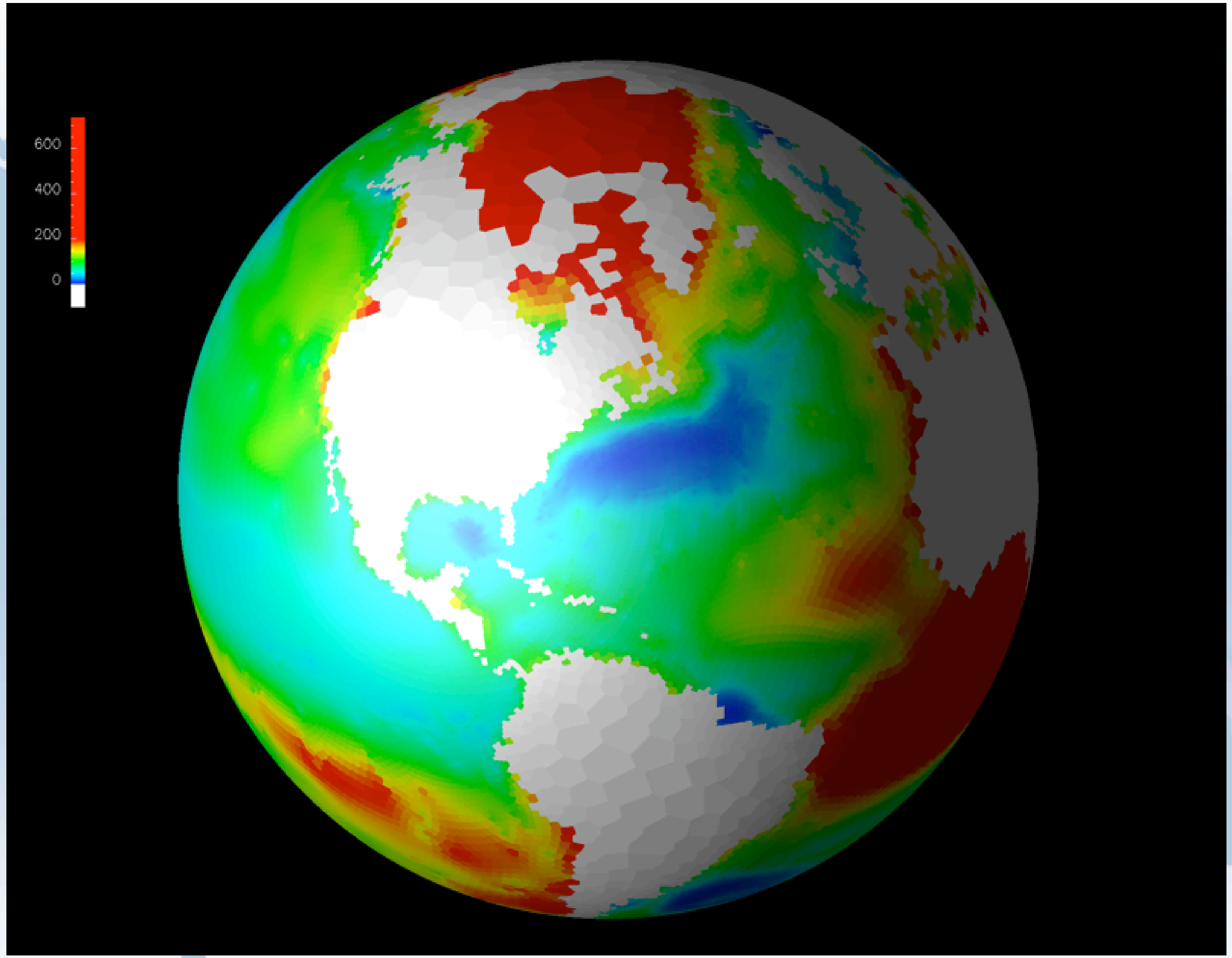
2) Use ssh variability as our proxy for generator density.



3) Iterate until convergence.

Generating a grid takes about 10 minutes on a desktop.

Average generator spacing in km ....



Average spacing is ~60 km.

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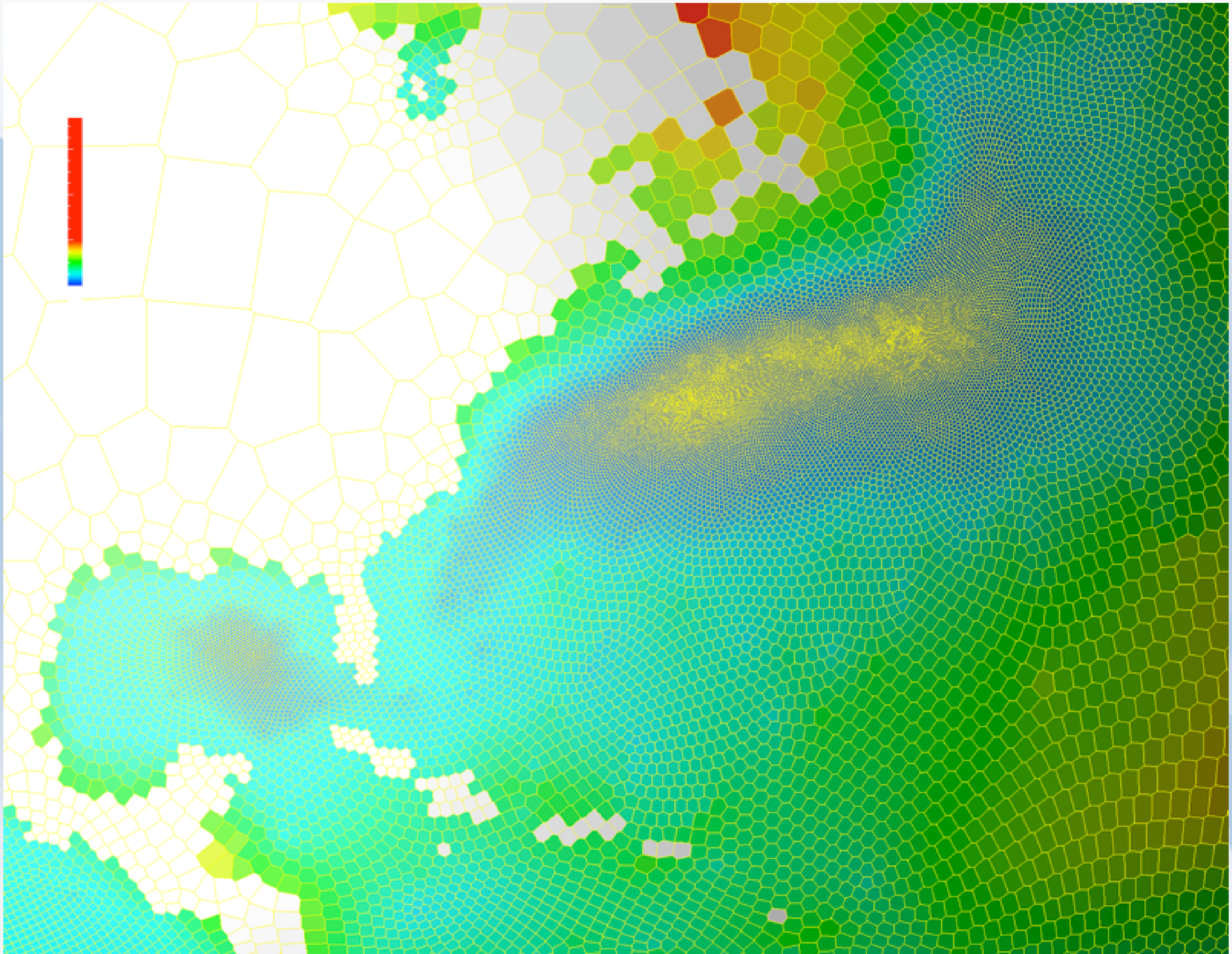
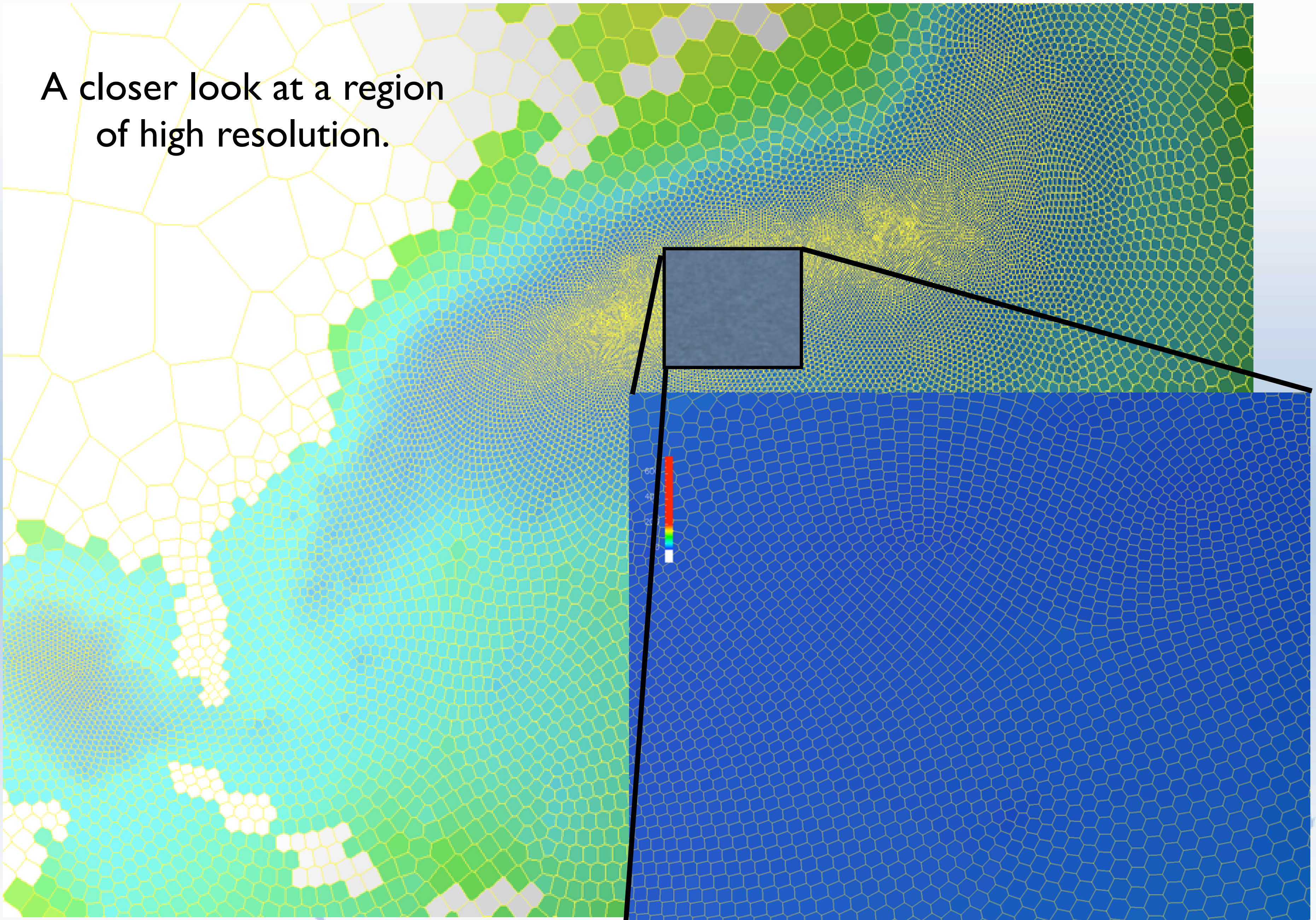
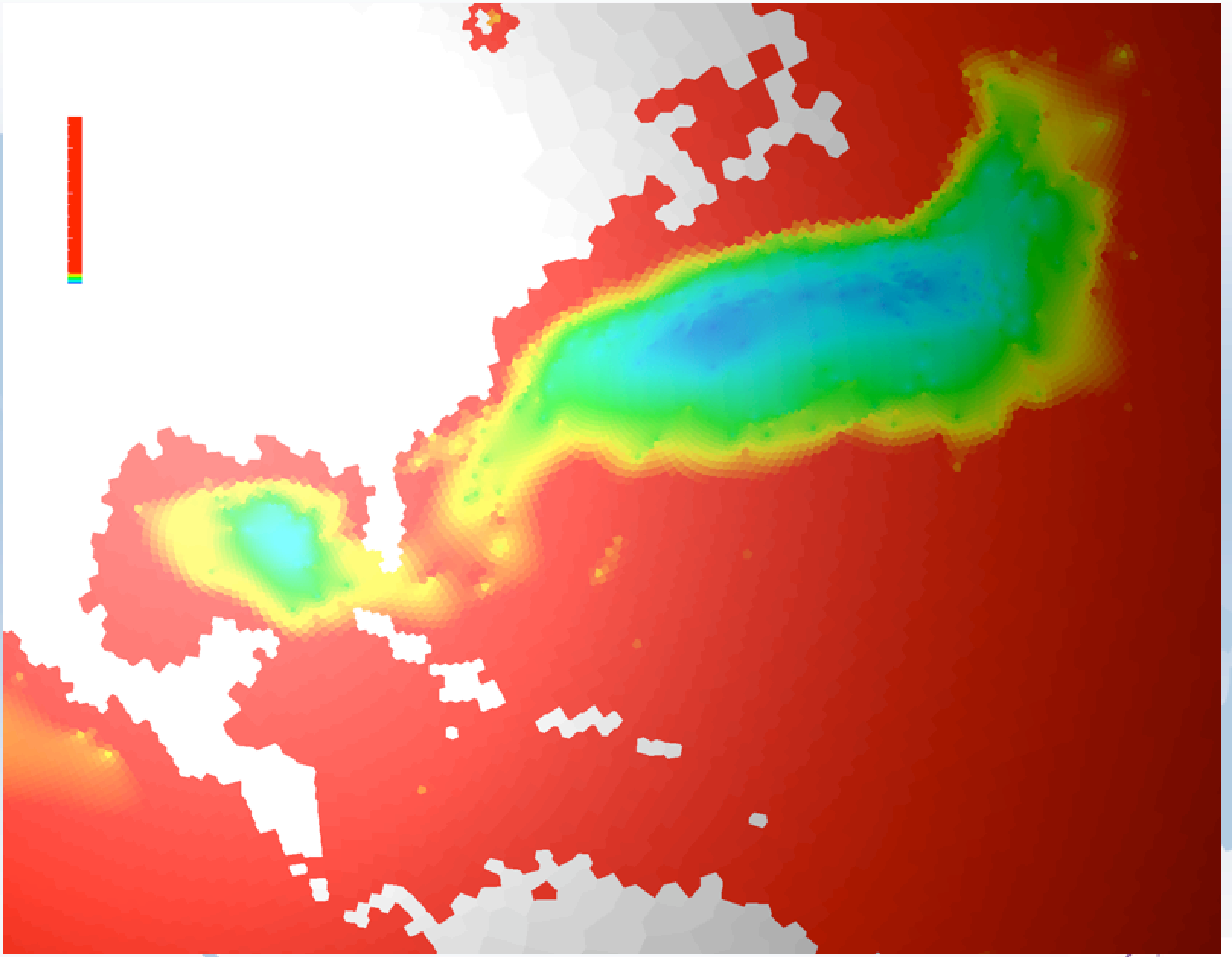
A closer look at the North Atlantic ....



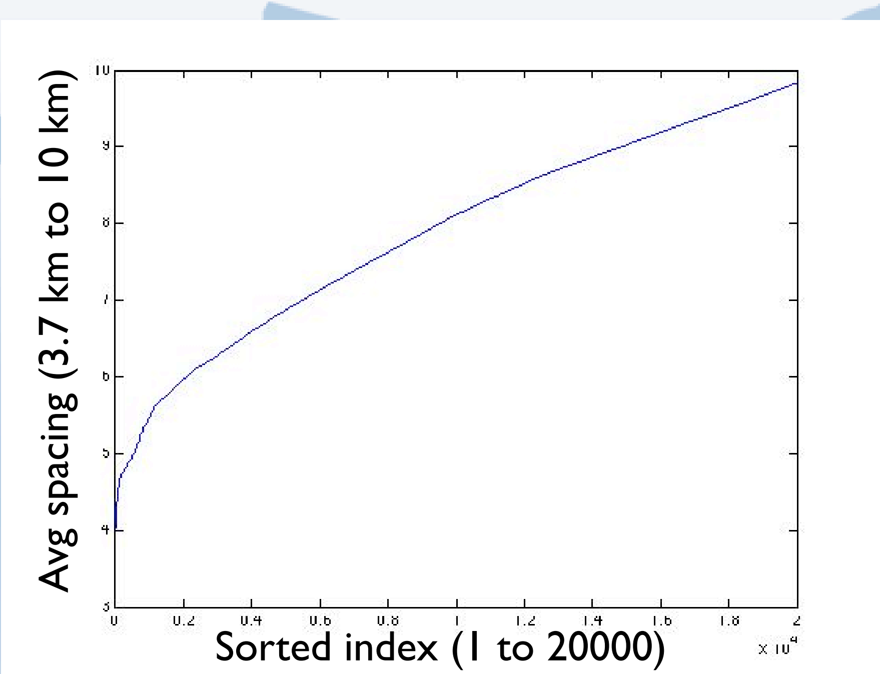
A closer look at a region of high resolution.

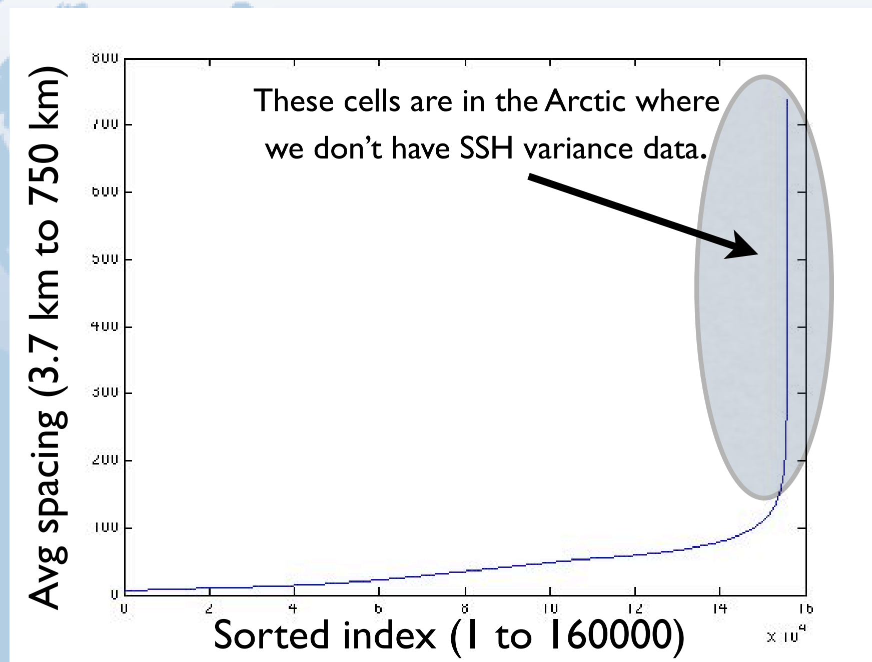


Region with less than 50 km spacing ...



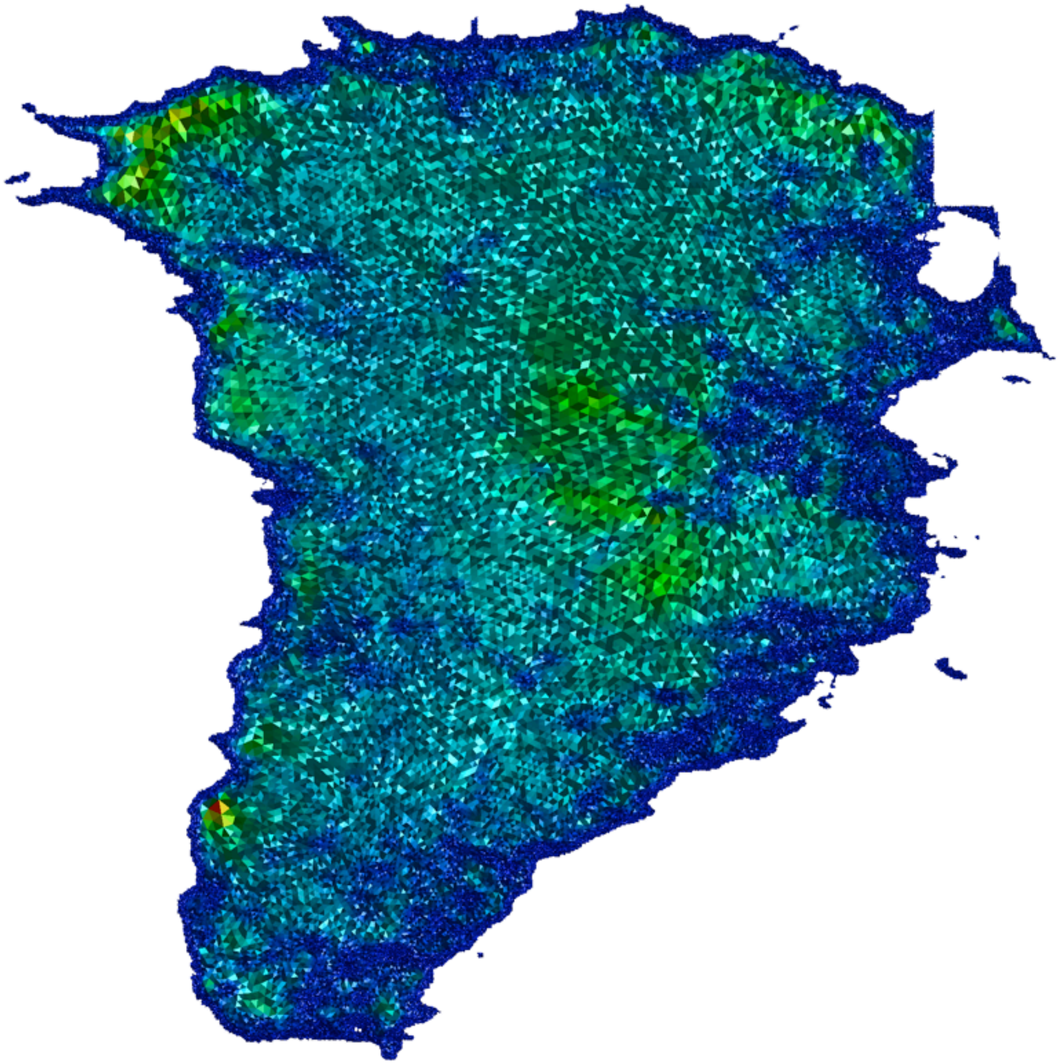
Distribution of grid spacing ....



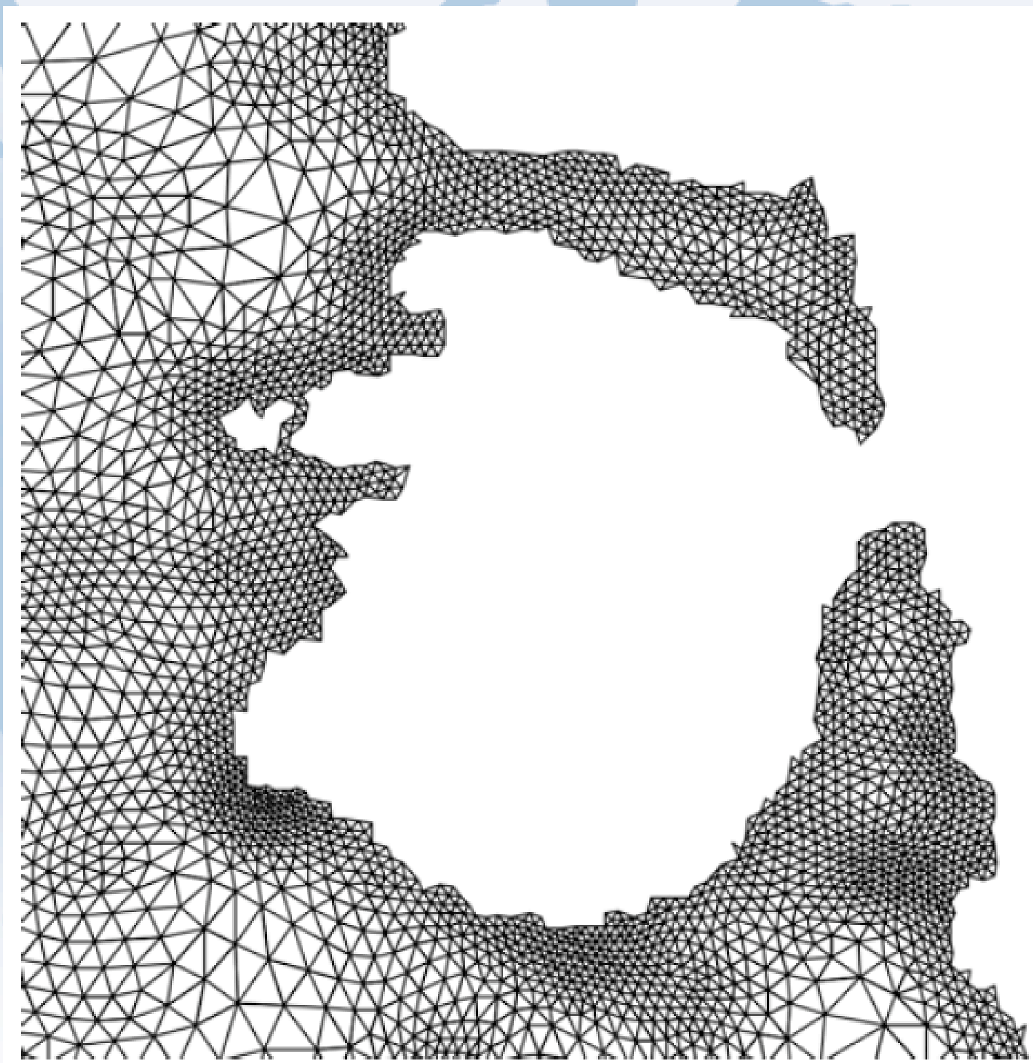


We think this grid is a good first try, but we will likely want to reduce the range of generator spacing. Keeping the min ~10 km and max ~100 km.

While the focus here has been on ocean modeling, other components of the climate system model could benefit significantly from SCVTs.



And dual grid looks attractive as well. Note that the grid is boundary conforming.



So where do we stand here ....

1) We have demonstrated that SCVT offer an attractive path to eddy-resolving ocean simulations.

2) Second-order finite-volume techniques that conserve energy or potential enstrophy already exist for these grids.

3) These grids offer significant savings in computational burden. Our preliminary estimate is a factor of 10 in savings.

4) We need high-performance computing tools to manage these unstructured grids, such as Global Array Toolkit.

5) We need to work on the density proxy for SCVT and the trade-off between smoothness and minimizing degrees of freedom.

Extra Slides

We have those methods already developed for the uniform SCVT.

1) all first-order quantities conserved exactly.
2) div-grad vector identity holds exactly.
3) curl(grad) vector identity holds exactly.
4) monotone, compatible transport scheme exists.

These properties carry over unaltered to (suitably regular) SCVT.

Generalization to any SCVT seems likely.